2/8/17

"The most difficult thing is the decision to act, the rest is merely tenacity."-Emelia Earhart

HW: "Even and Odd Functions" homework section

Test 1 on Thursday 2/16

AIM: How do we tell if a function is Even or Odd?

Warm Up:

inside (opposite)

1. The quadratic function $f(x)$ has a turning point at $(5,-8)$. If $g(x)=f(x+7)-3$, then at which of the following does $g(x)$ have a turning point?

-7 -3

$(-2,-11)$

left 7 subtract 7

down 3 subtract 3

(1) $(-2,-11)$

(3) $(-7,-3)$

(2) $(12,-11)$

(4) $(12,-5)$

1. The quadratic function $g(x)$ has a turning point at $(-12, 8)$. Where would the quadratic function $f(x) = g(4x)$ have a turning point?

(1) $(-48, 32)$ (3) $(-3, 8)$

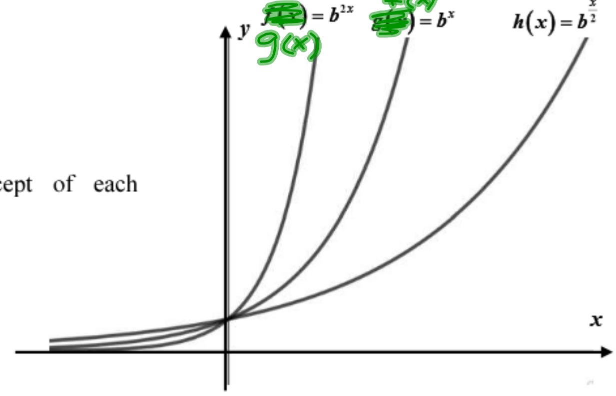
(2) $(-48, 8)$ (4) $(-3, 2)$

The graph of g has been horizontally compressed by a factor of 4. So, the x-coordinate of the turning point will be divided by 4.

(3)

2. The three exponential graphs shown below represent the function $f(x) = b^x$, $g(x) = b^{2x}$ and $h(x) = b^{\frac{x}{2}}$, for some $b > 1$.

(a) Label each with its correct equation.

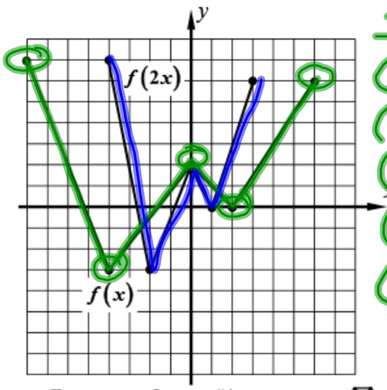


(b) Algebraically, show that the y-intercept of each function is the same.

$f(0) = b^0 = 1$
$g(0) = b^{2(0)} = b^0 = 1$
$h(0) = b^{\frac{0}{2}} = b^0 = 1$

3. The graph of $f(x)$ is shown on the grid below. Sketch a graph of $f(2x)$ on the same set of axes.

The graph of $f(x)$ will be compressed by a factor of 2 to produce the graph of $f(2x)$. Take each of the major points on $f(x)$ and simply divide its x-coordinate by 2.



$f(x)$	$f(2x)$
(-4,-3)	(-2,-3)
(0,2)	(0,2)
(2,0)	(1,0)
(6,6)	(3,6)
(-8,7)	(-4,7)

State the domain of the two functions:

Domain of $f(x)$: $\boxed{-8 \le x \le 6}$ [-8, 6]

Domain of $f(2x)$: $\boxed{-4 \le x \le 3}$

5. We've seen repeatedly that a horizontal dilation does not alter the graph's y-intercept. Given the function $f(x)$ and $g(x)=f(kx)$, can you determine an algebraic argument for why $f(x)$ and $g(x)$ must have the same y-intercepts? (Hint: Think about how we **always** find the y-intercept of any function). x=0

Plug in 0 for all x values.

f(0) and g(0)=f(0k)=f(0)

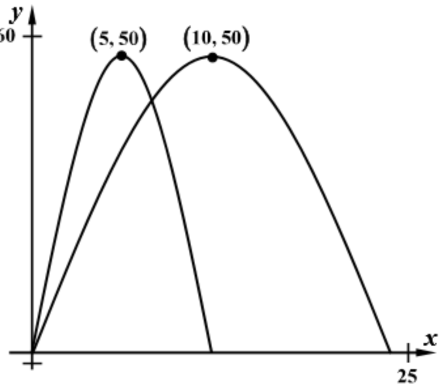
6. If the function $f(x)$ has a domain of $-2 \le x \le 8$ and a range of $-4 \le y \le 6$ and the function $g(x)$ is defined by the formula $g(x)=5f(2x)$ then what are the domain and range of g? Explain your thought process.

Domain: $\frac{-2}{2} \le x \le \frac{8}{2} \Rightarrow -1 \le x \le 4$ **Range:** $-4 \cdot 5 \le y \le 6 \cdot 5 \Rightarrow -20 \le y \le 30$

4. An arch is to be constructed so that its shape follows the curve $y=-\frac{1}{2}x^2+10x$, where x measures the horizontal distance along the ground and y measures the vertical height of the arch above the ground, both in units of feet. The general graph of this arch is shown below.

(a) Based on this equation, what is the height of the arch at the turning point? Show the work that leads to your answer.

$$x=\frac{-(10)}{2\left(-\frac{1}{2}\right)}=\frac{-10}{-1}=10 \Rightarrow y=-\frac{1}{2}(10)^2+10(10)=50 \text{ feet}$$



(b) If a second arch was to be created that had the same height, but only half the width, determine an equation for this arch based on our work in this lesson.

$$y=-\frac{1}{2}(2x)^2+10(2x)=-\frac{1}{2}\cdot 4x^2+20x$$
$$y=-2x^2+20x$$

(c) Choosing an appropriate graphing window based on (a), graph the second arch on the axes above. Label your graphing window. Use your calculator to determine the new turning point and label both points on the graphs.

Reasoning

5. We've seen repeatedly that a horizontal dilation does not alter the graph's y-intercept. Given the function $f(x)$ and $g(x)=f(kx)$, can you determine an algebraic argument for why $f(x)$ and $g(x)$ must have the same y-intercepts? (Hint: Think about how we **always** find the y-intercept of any function).

The y-intercept is always the output of the function when the input is $x=0$. So, the y-intercept of $f(x)$ is whatever the value of $f(0)$ is. But, if $g(x)=f(kx)$, then the y-intercept of $g(x)$ is $g(0)=f(k\cdot 0)=f(0)$, which is just the y-intercept of $f(x)$ again.

6. If the function $f(x)$ has a domain of $-2\le x\le 8$ and a range of $-4\le y\le 6$ and the function $g(x)$ is defined by the formula $g(x)=5f(2x)$ then what are the domain and range of g? Explain your thought process.

The graph of $g(x)$ would be produced by horizontally compressing the graph of $f(x)$ by a factor of 2 and then stretching it vertically by a factor of 5. So:

Domain: $\frac{-2}{2}\le x\le\frac{8}{2}\Rightarrow -1\le x\le 4$ **Range:** $-4\cdot 5\le y\le 6\cdot 5\Rightarrow -20\le y\le 30$

Recall that functions are simply rules that convert inputs or x values into outputs or y.

EVEN AND ODD FUNCTIONS

A function is known as **even** if $f(-x) = f(x)$ for every value of x in the domain of $f(x)$ (for all x values)

A function is known as **odd** if $f(-x) = -f(x)$ every value of x in the domain of $f(x)$.

***Exercise* #1:** Look at the definitions above and try to determine what they say about the inputs and outputs for these types of functions then write down your interpretation on the lines below. Remember that $f(x) = y$.

1. Even Functions: -x or x will result in the same y-value. ex $f(3)=5$ $f(-3)=5$

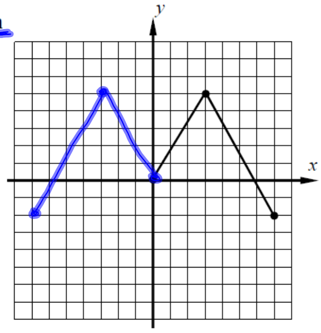
2. Odd Functions: if we switch the sign on x the result will switch the sign of y.

Ex $(2, 3) \longrightarrow (-2, -3)$

Let's take a look at **even** and **odd** functions first from a graphical standpoint.

***Exercise* #2:** Consider the **partial graph** of the function $f(x)$ shown twice below. Sketch the other half of the function if in (a) $f(x)$ is **even** and in (b) $f(x)$ is **odd**. The three coordinate pairs are listed to help you plot.

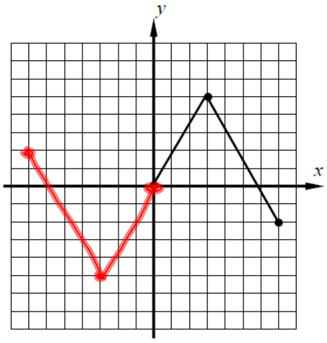
(a) **even**



$(0,0)$, $(3,5)$, $(7,-2)$

(0,0) (-3,5) (-7,-2)

(b) **odd**



$(0,0)$, $(3,5)$, $(7,-2)$

(0,0) (-3,-5) (-7,2)

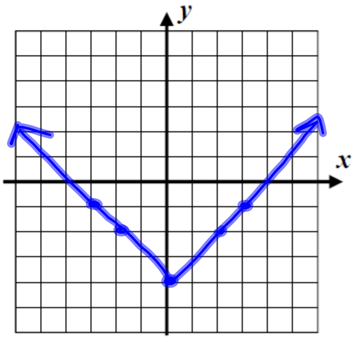
(c) Describe the symmetry of the **even** graph and the **odd** graph. Use as technically correct terminology as you can from your studies in Geometry.

EVEN: Reflect over y-axis

ODD: Rotation of 180°
Reflection over the origin

***Exercise* #3:** Consider the function $f(x) = |x| - 4$.

(a) Evaluate this function for a variety of opposite input pairs. What type (even, ~~odd,~~ or neither) does f appear to be?



$f(3) = |3| - 4 = -1$

$f(-3) = |-3| - 4 = -1$

$f(-2) = |-2| - 4 = -2$

$f(2) = |2| - 4 = -2$

(b) Sketch $f(x)$ on the grid below *without* the use of your calculator. Does it have the correct symmetry for your choice in (a)?

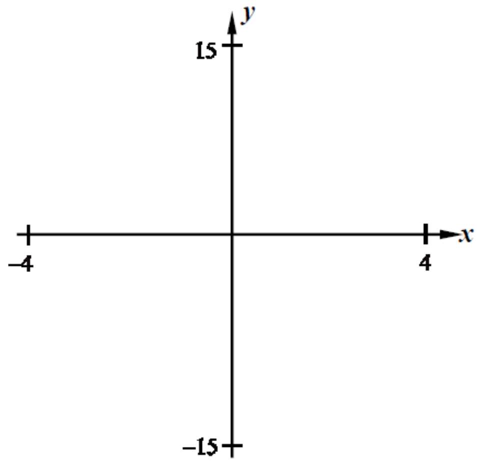
even
b/c it reflects
over the
y-axis.

***Exercise* #4:** Let's investigate $g(x) = x^3 - 4x$.

(a) Use your calculator's table option to fill in the following table. What type of function is this. Explain.

x	$g(x)$
-3	
-2	
-1	
0	
1	
2	
3	

(b) Sketch a graph of $g(x)$ using your calculator and the window indicated.



***Exercise* #5:** Is the simple exponential function $f(x) = 2^x$ odd, even, or neither? Support your argument with numerical evidence.